

Sudden Change of Quantum Discord under Single Qubit Noise

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We show that the sudden change of quantum correlation can occur even when only one part of the composite entangled state is exposed to a noisy environment. Our results are illustrated through the action of different noisy environments individually on a single qubit of quantum system. Composite noise on the whole of the quantum system is thus not the necessarily condition for the occurrence of sudden transition for quantum correlation.

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I. INTRODUCTION

Quantum entanglement, a kind of nonclassical correlation in quantum world, is a fundamental concept of quantum mechanics [1, 2]. It is well accepted that entanglement plays a crucial role and is the invaluable resource in quantum computation and quantum information [3, 4]. Recently, it is realized that entanglement is not the only aspect of quantum correlations, and the nonclassical correlations other than entanglement may also play fundamental roles in quantum information processing [5–7]. Among several measures of quantum correlations, the so called quantum discord, introduced by Olliver and Zurek [6] and also by Henderson and Vedral [8], has been receiving a great deal of attentions [9–29].

Quantum discord, as an important supplementary to quantum entanglement, is found to be present in the deterministic quantum computation with one qubit (DQC1) while the entanglement is vanishing [21, 30]. On the other hand, quantum discord is similar as entanglement, for example, it can be considered as a resource in some quantum information processing protocols [31, 32]. Quantum discord can also be related directly with quantum entanglement [33, 34]. One fundamental point is perhaps that quantum discord coincides quantum entanglement for pure state. However, besides different roles in DQC1, there are some other fundamental points which are different for quantum discord and quantum entanglement. One of the key differences is that quantum entanglement is non-increasing under the local operations and classical communication (LOCC), while quantum discord can increase under a local quantum channel acting on a single side of the studied bipartite state [35, 36]. This is surprising since it is generally believed that quantum correlations can only decrease generally under local quantum operations even classical communication is allowed. Here let us note that various quantum correlations, including quantum discord and quantum entanglement, are invariant under local unitary operations by definition.

To be more explicit, entanglement can increase only when coherent operations are applied. This is similar as in classical case, we know that classical correlation can increase when classical communication is allowed, which apparently

involves two parties. In comparison, quantum discord which describes the quantumness of correlations, can increase under one-side local quantum operations. In this paper, we will add more evidences to show that quantum discord may possess some properties under a single local quantum channel.

We know that contrary to the entanglement sudden death (ESD) [37–39], the behaviors of quantum discord under the Markovian environments decays exponentially and disappears asymptotically [13, 40]. But the investigation recently shows that the decay rates of quantum correlation may have sudden changes [15, 26] under composite noises. We already find some evidences showing that one side quantum channel is already enough for the occurrence of some phenomena for quantum discord, one may wonder whether the discord sudden change can occur when only one qubit of quantum system is subjected to a noisy environment while leaving the other subsystem free of noise? In this article, we investigate the dynamics of quantum discord of two qubit X shape state with only one particle exposed to noise. Our results show that composite noises are not necessary for the sudden change of quantum correlation, a single side of quantum channel is enough.

II. CLASSICAL AND QUANTUM CORRELATIONS OF X SHAPE STATES

It is widely accepted that the total correlation of a bipartite system ρ_{AB} is measured by the quantum mutual information defined as

$$I(\rho_{AB}) = S(\rho_A) + S(\rho_B) - S(\rho_{AB}),$$

where ρ_A and ρ_B are the reduced density matrices of ρ_{AB} , and $S(\rho) = -\text{Tr}\{\rho \log_2 \rho\}$ is the von Neumann entropy. Classical correlation [8] is defined as

$$C(\rho_{AB}) = \max_{B_i^\dagger B_i} S(\rho_A) - \sum_i p_i S(\rho_A^i),$$

where $B_i^\dagger B_i$ is a POVM performed on the subsystem B, $p_i = \text{Tr}_{AB}(B_i \rho_{AB} B_i^\dagger)$, and $\rho_A^i = \text{Tr}_B(B_i \rho_{AB} B_i^\dagger)/p_i$ is the post-measurement state of A after obtaining the outcome on B. Then quantum discord which quantifies the quantum correlation is given by

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$$Q(\rho_{AB}) = I(\rho_{AB}) - C(\rho_{AB}).$$

Consider the following Bell-diagonal states [12]

$$\rho = \frac{1}{4} \left(I \otimes I + \sum_{j=1}^3 c_j \sigma_j \otimes \sigma_j \right), \quad (1)$$

where I is the identity operator on the subsystem, $\sigma_j, j = 1, 2, 3$, are the Pauli operators, $c_j \in \mathbb{R}$ and such that the eigenvalues of ρ satisfying $\lambda_i \in [0, 1]$. The states in Eq. (1) represents a considerable class of states including the Werner states ($|c_1| = |c_2| = |c_3| = c$) and Bell ($|c_1| = |c_2| = |c_3| = 1$) basis states. The mutual information and classical correlation of the state ρ in Eq. (1) are given by [12]

$$I(\rho) = 2 + \sum_{i=0}^3 \lambda_i \log_2 \lambda_i,$$

$$C(\rho) = \frac{1-c}{2} \log_2(1-c) + \frac{1+c}{2} \log_2(1+c),$$

where $c = \max\{|c_1|, |c_2|, |c_3|\}$, and then the quantum discord of state ρ is given as

$$Q(\rho) = I(\rho) - C(\rho)$$

$$= 2 + \sum_{i=1}^4 \lambda_i \log_2 \lambda_i - \frac{1-c}{2} \log_2(1-c) - \frac{1+c}{2} \log_2(1+c).$$

A generalization of quantum discord from Bell-diagonal states to a class of X shape state is given in Ref. [19] recently with state in form,

$$\rho' = \frac{1}{4} \left(I \otimes I + \mathbf{r} \cdot \boldsymbol{\sigma} \otimes I + I \otimes \mathbf{s} \cdot \boldsymbol{\sigma} + \sum_{i=1}^3 c_i \sigma_i \otimes \sigma_i \right) \quad (2)$$

where $\mathbf{r} = (0, 0, r)$, $\mathbf{s} = (0, 0, s)$, one can find that ρ' reduces to ρ when $r = s = 0$. The mutual information and classical correlation of state ρ' are given by

$$I(\rho') = S(\rho'_A) + S(\rho'_B) - S(\rho'),$$

$$C(\rho') = S(\rho'_A) - \min\{S_1, S_2, S_3\}.$$

Here S_1, S_2, S_3 are shown in [19], and $f(t)$ is defined as $f(t) = -\frac{1+t}{2} \log_2(1+t) - \frac{1-t}{2} \log_2(1-t)$. Then quantum discord, the quantum correlation of state ρ' , is given by

$$Q(\rho') = I(\rho') - C(\rho')$$

$$= S(\rho'_B) - S(\rho') + \min\{S_1, S_2, S_3\}.$$

It is difficult to calculate quantum discord in general case since the optimization should be taken. However, the analytical expression of quantum discord about Bell-diagonal states is available [12] which provides a convenient method in studying the dynamics of quantum correlation in case the studied states satisfy this special form [15, 26]. In this paper, based on an analytical expression of quantum correlation which generalize the discord from Bell-diagonal states to a class of X shape states [19], we can investigate dynamics of quantum discord with more kinds of quantum noises. In particular, the analytical discord can be found for cases with one side quantum channel. As a result, we can study three different kinds of quantum channels, amplitude, dephasing, and depolarizing which act on the first qubit of a class of two-qubit X states. We next consider those three quantum channels respectively.

III. AMPLITUDE NOISE

Amplitude damping or amplitude noise which is used to characterize spontaneous emission describes the energy dissipation from a quantum system. The Kraus operators for a single qubit are given by [37]

$$E_0 = \begin{pmatrix} \eta & 0 \\ 0 & 1 \end{pmatrix}, E_1 = \begin{pmatrix} 0 & 0 \\ \sqrt{1-\eta^2} & 0 \end{pmatrix},$$

where $\eta = e^{-\frac{\tau}{2}}$ and τ is the amplitude decay rate, t is time. We consider the case that the first qubit is through this quantum channel. So the Kraus operators for the whole system, with amplitude noise acting only on the first qubit, are given by

$$K_{1a} = \begin{pmatrix} \eta & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

$$K_{2a} = \begin{pmatrix} 0 & 0 \\ \sqrt{1-\eta^2} & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Let $\mathcal{E}(\cdot)$ represents the operator of the noise environment. With the time-dependent Kraus operator matrix of amplitude noise acting on the first qubit of state ρ , we have

$$\begin{aligned}\varepsilon_a(\rho) &= K_{1a}\rho K_{1a}^\dagger + K_{2a}\rho K_{2a}^\dagger \\ &= \frac{1}{4} \begin{pmatrix} \eta^2(1+c_3) & 0 & 0 & \eta(c_1-c_2) \\ 0 & -\eta^2(-1+c_3) & \eta(c_1+c_2) & 0 \\ 0 & \eta(c_1+c_2) & (2-\eta^2-\eta^2c_3) & 0 \\ \eta(c_1-c_2) & 0 & 0 & (2-\eta^2+\eta^2c_3) \end{pmatrix}.\end{aligned}$$

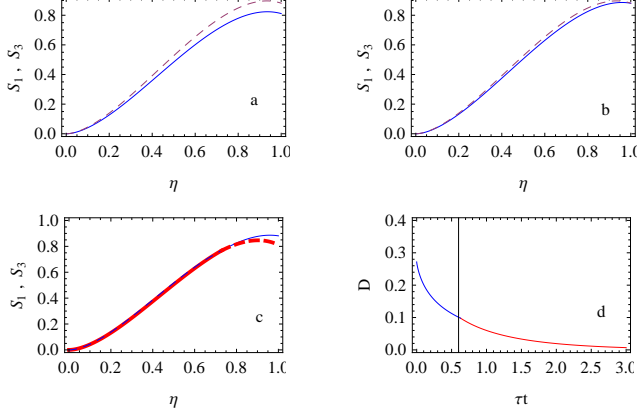


FIG. 1: S_1 (dashed line) and S_3 (solid line) of Eq. (3), Eq. (3): (a) $c_2 = 0.5, c_3 = 0.4$, $S_3 < S_1$ when $\eta \in (0, 1)$; (b) $c_2 = c_3 = 0.4$, $S_3 < S_1$ when $0 < \eta < 1$; (c) $c_1 = 0.1, c_2 = 0.4, c_3 = 0.5$, $S_3 < S_1$ when $\eta \in (0, 0.73)$ and $S_1 < S_3$ when $\eta \in (0.73, 1)$; (d) Quantum discord of $\varepsilon_a(\rho)$ with the same parameters as in situation (c), the decay rate of quantum discord sudden changes at $\tau t = 0.63$.

One can readily find that the state $\varepsilon_a(\rho)$ has the same form with state in Eq.(2)

$$\varepsilon_a(\rho) = \frac{1}{4} (I \otimes I + r(t)\sigma_3 \otimes I + s(t)I \otimes \sigma_3 + \sum_{i=1}^3 c_i(t)\sigma_i \otimes \sigma_i),$$

here $r(t) = \eta^2 - 1, s(t) = 0, c_1(t) = \eta c_1, c_2(t) = \eta c_2, c_3(t) = \eta^2 c_3$, then by the formula given by [19], we have that

$$\begin{aligned}S_1 &= 1 + \frac{1}{2}f(\eta^2 - 1 + \eta^2 c_3) + \frac{1}{2}f(\eta^2 - 1 - \eta^2 c_3), \\ S_2 &= 1 + f(\sqrt{(\eta^2 - 1)^2 + (\eta c_1)^2}), \\ S_3 &= 1 + f(\sqrt{(\eta^2 - 1)^2 + (\eta c_2)^2}).\end{aligned}$$

We find that it is always true that $S_3 < S_2$ (or $S_2 < S_3$) if $c_2 > c_1$ (or $c_1 > c_2$), so we just need to compare S_1 with S_3 (or S_2) to obtain the minimum of $\{S_1, S_2, S_3\}$. We note, $S_3 \leq S_1$, always happens when $c_2 \geq c_3$, in comparison, the minimum of $\{S_1, S_2, S_3\}$ transfers from S_3 to S_1 when $c_2 < c_3$. We show this in Fig.(1). The sudden change of quantum discord happens when $c_2 < c_3$ in the initial state ρ with this kind of noise environment.

IV. PHASE NOISE

Phase noise or phase damping channel describes a quantum noise with loss of quantum phase information without loss of energy. The Kraus operators of this noise for single qubit are given by [4, 41]

$$K_0 = \begin{bmatrix} 1 & 0 \\ 0 & \gamma \end{bmatrix}, \quad K_1 = \begin{bmatrix} 0 & 0 \\ 0 & \sqrt{1-\gamma^2} \end{bmatrix},$$

where $\gamma = e^{-\frac{\tau}{\tau}}$ and τ denotes transversal decay rate. The operators of phase noise acting on the first qubit of state ρ , so we have $K_{1p} = K_1 \otimes I_2, K_{2p} = K_2 \otimes I_2$, then one can find,

$$\begin{aligned}\varepsilon_p(\rho) &= K_{1p}\rho K_{1p}^\dagger + K_{2p}\rho K_{2p}^\dagger \\ &= \frac{1}{4} \begin{pmatrix} 1+c_3 & 0 & 0 & \gamma(c_1-c_2) \\ 0 & 1-c_3 & \gamma(c_1+c_2) & 0 \\ 0 & \gamma(c_1+c_2) & 1-c_3 & 0 \\ \gamma(c_1-c_2) & 0 & 0 & 1+c_3 \end{pmatrix} \\ &= \frac{1}{4} (I + \gamma c_1 \sigma_1 \otimes \sigma_1 + \gamma c_2 \sigma_2 \otimes \sigma_2 + c_3 \sigma_3 \otimes \sigma_3) \quad (3)\end{aligned}$$

Comparing Eq. (3) with Eq. (2), we can easily obtain the classical correlation and quantum correlation of state ρ under phase noise acting on the first qubit,

$$C(\varepsilon_p(\rho)) = \frac{1-\chi}{2} \log_2(1-\chi) + \frac{1+\chi}{2} \log_2(1+\chi), \quad (4)$$

$$\begin{aligned}Q(\varepsilon_p(\rho)) &= I(\varepsilon_p(\rho)) - C(\varepsilon_p(\rho)) \\ &= 2 + \sum_{i=1}^4 \lambda_i \log_2 \lambda_i - \frac{1-\chi}{2} \log_2(1-\chi) - \frac{1+\chi}{2} \log_2(1+\chi),\end{aligned} \quad (5)$$

where $\chi = \max\{|\gamma c_1|, |\gamma c_2|, |c_3|\}$, and $\{\lambda_i\}$ are the eigenvalues of $\varepsilon_p(\rho)$. If $|c_3| \geq \max\{|c_1|, |c_2|\}, \chi$ in Eq.(4) and Eq.(5) will equal to $|c_3|$, and the classical correlation $C(\varepsilon_p(\rho))$ remains unaffected, while the quantum correlation $Q(\varepsilon_p(\rho))$ decays monotonically. If $\max\{|c_1|, |c_2|\} \geq |c_3|$ and $|c_3| \neq 0$, the dynamics of classical correlation $C(\varepsilon_p(\rho))$ and quantum correlation $Q(\varepsilon_p(\rho))$ have a sudden change at $t_0 = -\frac{2}{\tau} \log_2 \left| \frac{c_3}{\max\{c_1, c_2\}} \right|$. In Fig. 2, we depict the dynamic of quantum discord of ρ under single qubit phase noise with different $\{c_i\}$. It is shown that sudden change of quantum discord can occur when phase noise act only on one part of a two-qubit quantum state.

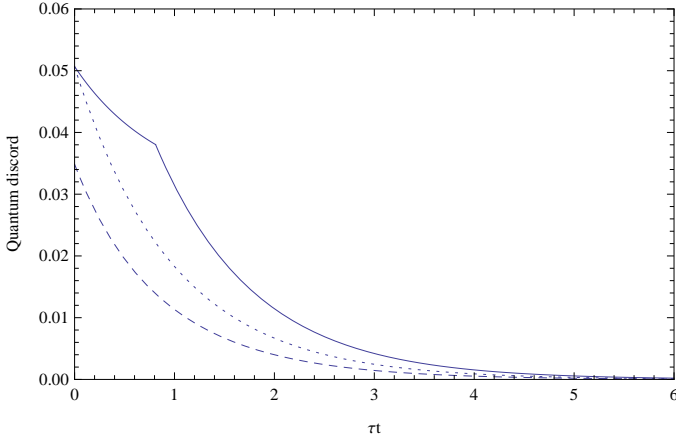


FIG. 2: Quantum discord of ρ under phase noise acting on the first qubit of the quantum system. (1) $c_1 = 0.1, c_2 = 0.2, c_3 = 0.3$ (dotted line). (2) $c_1 = 0.1, c_2 = 0.4, c_3 = 0.2$ (solid line). (3) $c_1 = c_2 = 0.2, c_3 = 0$ (dashed line). The sudden change happens only at situation (2).

V. DEPOLARIZING NOISE

The depolarizing noise is an important type of quantum noise that take a single qubit into completely mixed state $I/2$ with probability p and leave itself untouched with probability $1 - p$. The operators for single qubit depolarizing noise are given by [4]

$$D_1 = \sqrt{1-p} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad D_2 = \sqrt{\frac{p}{3}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

$$D_3 = \sqrt{\frac{p}{3}} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad D_4 = \sqrt{\frac{p}{3}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Where $p = 1 - e^{-\tau t}$, then we have the operators $\{K_{id}\}$ acting on the first qubit of a composite system $K_{1d} = D_1 \otimes I_2$, $K_{2d} = D_2 \otimes I_2$, $K_{3d} = D_3 \otimes I_2$, $K_{4d} = D_4 \otimes I_2$. The two-qubit system under the depolarizing noise acting on the first qubit of quantum state ρ is given as,

$$\begin{aligned} \varepsilon_d(\rho) &= \sum_{i=1}^4 K_{id} \rho K_{id}^\dagger \\ &= \frac{1}{4} \begin{pmatrix} 1 + \left(1 - \frac{4p}{3}\right)c_3 & 0 & 0 & \left(1 - \frac{4p}{3}\right)(c_1 - c_2) \\ 0 & 1 - \left(1 - \frac{4p}{3}\right)c_3 & \left(1 - \frac{4p}{3}\right)(c_1 + c_2) & 0 \\ 0 & \left(1 - \frac{4p}{3}\right)(c_1 + c_2) & 1 - \left(1 - \frac{4p}{3}\right)c_3 & 0 \\ \left(1 - \frac{4p}{3}\right)(c_1 - c_2) & 0 & 0 & 1 + \left(1 - \frac{4p}{3}\right)c_3 \end{pmatrix}. \end{aligned} \quad (6)$$

Comparing Eq.(6) with Eq.(2), we obtain that

$$\varepsilon_d(\rho) = \frac{1}{4} \left(I \otimes I + r(t) \sigma_3 \otimes I + s(t) I \otimes \sigma_3 + \sum_{i=1}^3 c_i(t) \sigma_i \otimes \sigma_i \right), \quad (7)$$

here $c_1(t) = \left(1 - \frac{4p}{3}\right)c_1$, $c_2(t) = \left(1 - \frac{4p}{3}\right)c_2$, $c_3(t) = \left(1 - \frac{4p}{3}\right)c_3$. We find that the maximum of $\{c_i(t)\}$ is always decided by the maximum of $\{c_i\}$ which is fixed and is independent of time, thus there is no sudden change of decay rate for the classical and quantum correlations of state ρ under depolarizing noise. We show this in Fig.(3). Quantum discord decays to 0 at $\Gamma t = \log_2 4$, and this means state ρ has turned to a completely mixed state. We have also consider the case that state ρ' is under the depolarizing noise, and we find that the minimum of $\{S_i\}$ does not change from S_1 (or S_3) to S_3 (or S_1) under this kind of noise. Therefore, there is no sudden change for quantum discord of state ρ' under depolarizing noise, see Fig.(4).

VI. CONCLUSION

We have studied the quantum discord of the X class of quantum states under three different kinds of noise such as amplitude, dephasing and depolarizing in this article. We show that composite noise is not the necessary condition for the occurrence of sudden change of quantum discord and the sudden change can happen when the quantum noise act only on one qubit of the quantum system. Especially, the classical correlation can remain unaffected under the phase noise acting only on the first qubit of state ρ just like it occurs when ρ is exposed into a composite noise environment [15]. On the other hand, we should note that the single noise acting on only one of the qubits of an two-qubit quantum state is not sufficient for the happening of sudden change of quantum discord, the suitable state is also needed.

The properties of various quantum correlations are studied generally for different situations with both coherent and individual operations. Quantum discord can demonstrate some special phenomena which can happen with only one side of quantum operation. The results in this paper provide more evidences that quantum discord is different from the quantum entanglement. Experimentally, the bipartite qubits system with

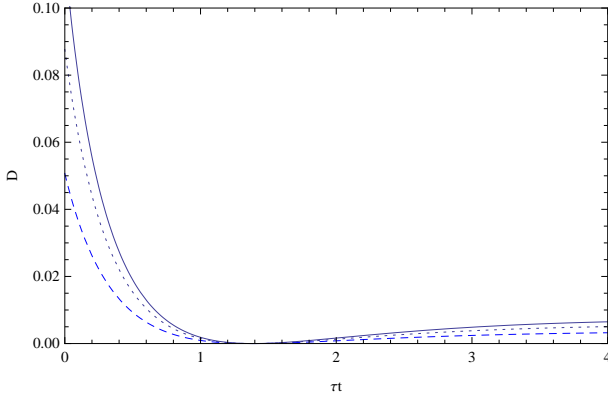


FIG. 3: Quantum discord of state ρ under depolarizing noise with (1) $c_1 = 0.1, c_2 = 0.2, c_3 = 0.3$ (dashed line), (2) $c_1 = 0.1, c_2 = 0.4, c_3 = 0.3$ (solid line) and (3) $c_1 = 0.3, c_2 = 0.2, c_3 = 0.2$ (dotted line) respectively. All of the three lines turn to 0 at $\Gamma t = \log_2 4$.

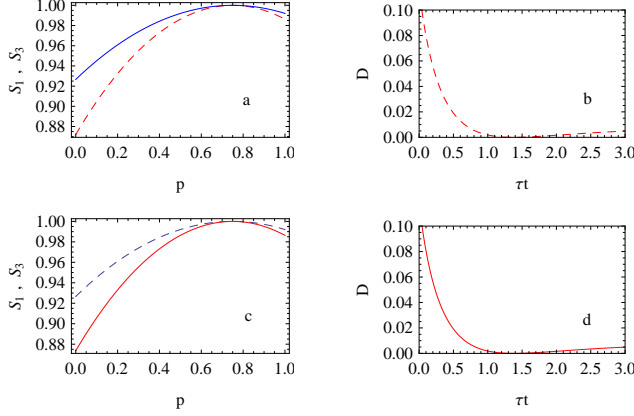


FIG. 4: Depolarizing noise acting on state ρ' : S_1 (dashed line) is always less than S_3 (solid line) in (a) $\{c_1 = 0.1, c_2 = 0.3, c_3 = 0.4, r = 0.1, s = -0.01\}$ and the quantum discord in this situation is shown in (b); S_3 (solid line) is always less than S_1 (dashed line) in (c) $\{c_1 = 0.1, c_2 = 0.4, c_3 = 0.3, r = 0.1, s = 0.01\}$, and we show quantum discord of this situation in (d).

various one side quantum channels can be realized in optical, nuclear magnetic resonance, superconducting qubits and solid states such as nitrogen-vacancy diamond systems. The behavior of sudden change in quantum discord should be observed explicitly.

We notice a related paper [42] in which the behaviors of quantum correlations under phase damping and amplitude damping channels acting only on the apparatus are considered. And the sudden change of discord is also observed which confirms our conclusion. On the other hand, there are some differences between our results and theirs. In [42], the maximal classical correlation, which appears as one part in definition of quantum discord, can be achieved by two different measurement projectors, i.e., in the σ_z basis or in the σ_x basis. In comparison, our results need to consider the minimum of S_1, S_2, S_3 as shown in Eq. (3) introduced in Ref.[19], so the explicit analytic expressions for the dynamics of quantum correlation under different kinds of noise can be found.

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